

Radicales - Propiedades básicas

1.- Relació entre arrel i potència: $\sqrt[n]{a} = a^{\frac{1}{n}}$. Per extensió: $\sqrt[n]{a^m} = a^{\frac{m}{n}}$.

Exemples:

- $\sqrt[3]{a^5} = a^{\frac{5}{3}}$
- $\sqrt[5]{x^2 \cdot y} = (x^2 \cdot y)^{\frac{1}{5}} = (x^2)^{\frac{1}{5}} \cdot y^{\frac{1}{5}} = x^{\frac{2}{5}} \cdot y^{\frac{1}{5}}$

2.- Relació amb l'exponent negatiu: $\frac{1}{\sqrt[n]{a^m}} = \sqrt[n]{a^{-m}} = a^{-\frac{m}{n}}$ i a l'inrevés. $\frac{1}{\sqrt[n]{a^{-m}}} = \sqrt[n]{a^m} = a^{\frac{m}{n}}$.

Exemple: $\sqrt{3} = \frac{1}{\sqrt{3^{-1}}} = \frac{1}{3^{-\frac{1}{2}}}$

3.- Potència d'un radical: $(\sqrt[n]{a^p})^k = \sqrt[n]{a^{p \cdot k}}$ Exemple: $(\sqrt[4]{7})^3 = \sqrt[4]{7^3}$

4.- Arrel d'un radical: $\sqrt[n]{\sqrt[m]{a}} = \sqrt[n \cdot m]{a}$ Exemple: $\sqrt[3]{\sqrt[4]{5}} = \sqrt[3 \cdot 4]{5} = \sqrt[12]{5}$

Exemples 2: $\sqrt[2]{x \sqrt[3]{x}} = \sqrt[2]{\sqrt[3]{x \cdot x^3}} = \sqrt[2]{\sqrt[3]{x^4}} = \sqrt[6]{x^4} = \sqrt[3]{x^{\frac{4}{3}}}$

Operacions amb arrels:

Multiplicar y dividir:

- Si tienen los índices iguales se multiplican/dividen las bases: $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$ y $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$
- Si tienen bases iguales se suman los inversos de los índices
- Si tienen índices y bases diferentes: se busca mediante el mcm que tengan índices iguales.

Sacar un término de la raíz: Es necesario tener elevado el valor a la misma potencia que el índice de la raíz. $\sqrt{12} = \sqrt{2^2 \cdot 3} = 2 \cdot \sqrt{3}$

Sumar y restar: No se puede.

Sólo sacar factor común: $\sqrt{12} + \sqrt{3} = 2 \cdot \sqrt{3} + \sqrt{3} = \sqrt{3} (2 + 1) = 3\sqrt{3}$

Racionalització:

El joc consisteix en treure l'arrel del denominador. 2 tipus

1.- Una arrel al denominador:

Se multiplica numerador y denominador por el denominador:

$$\frac{3}{\sqrt[4]{2}} = \frac{3 \cdot \sqrt[4]{2^3}}{\sqrt[4]{2 \cdot 2^3}} = \frac{3\sqrt[4]{8}}{2}$$

2.- Una arrel amb altre que suma o resta al denominador:

Se multiplica numerador y denominador por el conjugado del denominador

$$\frac{2}{3 - \sqrt{5}} = \frac{2(3 + \sqrt{5})}{(3 - \sqrt{5}) \cdot (3 + \sqrt{5})} = \frac{2(3 + \sqrt{5})}{3^2 - \sqrt{5}^2} = \frac{2(3 + \sqrt{5})}{4} = \frac{3 + \sqrt{5}}{2}$$

MISCELANEA

Calcular :

1.-
$$\frac{5\sqrt{50} + 18\sqrt{32} - 6\sqrt{200}}{3\sqrt{2}}$$

2.-
$$\sqrt[3]{16} + \sqrt[3]{250} + 2\sqrt[3]{54}$$

3.-
$$(3\sqrt{7} + \sqrt{2})^2 - (2\sqrt{7} + \sqrt{2})(2\sqrt{7} - \sqrt{2})$$

4.-
$$(\sqrt{5} - 1)(\sqrt{2} + 1) - (\sqrt{5} + 1)(\sqrt{2} + 1) + \sqrt{2}$$

5.-
$$(2 + \sqrt{2})(2 - \sqrt{3}) - \sqrt{2}(2 - \sqrt{3})$$

6.-
$$(6\sqrt{7} - 5\sqrt{7} + 2\sqrt{7})(8\sqrt{2} - 5\sqrt{2})\sqrt{2}$$

7.-
$$(\sqrt{18} : \sqrt{9} + \sqrt{27} : \sqrt{9} - \sqrt{32} : \sqrt{8} - \sqrt{3} + 2)\sqrt{2}$$

D) Expresa como cociente y calcula:

1.-
$$\left(\sqrt[3]{\frac{27}{8}} - \sqrt[3]{\frac{125}{64}} + 3\sqrt[3]{\frac{27}{8}}\right) : \left(\sqrt[3]{\frac{64}{27}} + \sqrt{\frac{81}{64}}\right)$$

2.-
$$\sqrt{\frac{2(a^2 + b^2)^2}{c^2} - \frac{2(a^2 - b^2)^2}{c^2}}$$

3.-
$$\sqrt{ab} : \left(\sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}}\right)$$

4.-
$$\sqrt[3]{\frac{25^3}{64^3}} + \sqrt[3]{\frac{8^2}{27^2}} - \sqrt[3]{\frac{25^x}{125^x}}$$

E) Aplicando la propiedad de la raíz de una raíz calcular:

1.-
$$3\sqrt[3]{\sqrt{x}} - 2\sqrt[3]{\sqrt{x}} + 5\sqrt{x}$$

2.-
$$\sqrt[3]{\sqrt[4]{a^3}} + \sqrt[5]{\sqrt[4]{a^5}} - 2\sqrt[7]{\sqrt[4]{a^7}}$$

3.-
$$2\sqrt[3]{\sqrt[4]{ab}} + \sqrt[6]{\sqrt{ab}} - 4\sqrt[4]{\sqrt[3]{ab}} + \sqrt[6]{ab}$$

4.-
$$\sqrt[9]{\sqrt[5]{x^{11}}} \cdot \sqrt[3]{\sqrt[15]{x^{10}}} \cdot \sqrt{x}$$

5.-
$$\sqrt[9]{\sqrt{\sqrt{x^6}}} + \sqrt[3]{\sqrt[5]{\sqrt[3]{x^5}}} - \sqrt[3]{\sqrt[4]{\sqrt{x^6}}} - \sqrt[3]{\sqrt[15]{x^5}}$$

6.-
$$(x+y)\sqrt{\frac{x^2 + y^2}{x^2 + 2xy + y^2}} \quad (\sqrt{x^2 + y^2})$$

7.-
$$(\sqrt{3} + \sqrt{2})\sqrt{5 - 2\sqrt{6}} ; (2 + \sqrt{3})\sqrt{7 - 4\sqrt{3}}$$

8.-
$$\left(\frac{9^{x+\frac{1}{4}}\sqrt[4]{9^{-\frac{1}{2}} \cdot 3^x}}{3\sqrt{3^{-x}}}\right)^{\frac{1}{x}} ; \left[\frac{(x-1)}{3}\sqrt[3]{\frac{(x-1)^2}{3}}\right]^4$$

9.-
$$(2 + \sqrt{3})\sqrt{7 - 4\sqrt{3}} ; (7 + \sqrt{5})\sqrt{54 - 14\sqrt{5}}$$

F) Aplica la propiedad de cambio de índice

1.-
$$\sqrt[3]{a} \cdot \sqrt{a^3} \cdot \sqrt[5]{a} ; x^{-y}\sqrt{x^{x^2-y^2}} ; \frac{\sqrt[5]{x^2} \cdot \sqrt[3]{x^4}}{\sqrt[5]{x} \cdot \sqrt[3]{x^5}}$$

2.-
$$2\sqrt{a} - 3\sqrt[3]{a} + 5\sqrt[4]{a}$$

3.-
$$\frac{5\sqrt[3]{a^2b^2} \cdot 2\sqrt[3]{a^2b}}{2\sqrt{ab} \cdot 3\sqrt[4]{a^3b^3}}$$

4.-
$$\sqrt[3]{ab^2} \cdot a\sqrt{ab}\sqrt{ab} \cdot \sqrt[9]{a^5b^3}$$

5.-
$$\left[\frac{a\sqrt[3]{x\sqrt{ax}} \cdot x\sqrt[5]{a^3\sqrt{a^2x}} \cdot ax\sqrt[4]{a^3x^2}}{2x}\right]^2$$

6.-
$$\frac{a^3\sqrt{x} \cdot 2x \cdot \sqrt{\sqrt{a^3}}}{\sqrt[3]{x\sqrt{2x^3}} (\sqrt{2a})^3}$$

7.-
$$\frac{a^4\sqrt{x^5y^{17}} \cdot \sqrt[4]{b^4xy^9}}{\sqrt[4]{x^{13}y^9a^4} \cdot b \cdot \sqrt[4]{x^5y^5}}$$

8.-
$$\sqrt{2\sqrt{2\sqrt[3]{2\sqrt{2}}}} + \sqrt{3\sqrt[3]{3\sqrt{3\sqrt{3}}}}$$

Racionalizar:

1.- $\frac{1}{\sqrt{2}}; \frac{\sqrt{2}}{3\sqrt{3}}; \frac{5+\sqrt{18}}{\sqrt{2}}$

2.- $\frac{a}{\sqrt[3]{a}}; \frac{x}{\sqrt[4]{x}}; \frac{a}{\sqrt{x}}$

3.- $\frac{a\sqrt{a}}{\sqrt[3]{a^2}}; \frac{3}{\sqrt[5]{5}}$

4.- $\frac{6}{2+\sqrt{2}}; \frac{1}{3-\sqrt{2}}; \frac{\sqrt{5}}{\sqrt{5}-\sqrt{2}}$

5.- $\frac{\sqrt{2}-\sqrt{3}}{2+\sqrt{2}}; \frac{\sqrt{2}-\sqrt{2}}{\sqrt{2+\sqrt{2}}}; \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2+\sqrt{3}}}$

6.- $\left(\frac{2-\sqrt{2}}{1-\sqrt{2}}\right)^{\frac{1}{2}}$

7.- $\frac{2\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{3\sqrt{2}}{\sqrt{5}-\sqrt{2}} + \frac{2}{\sqrt{7}-\sqrt{5}}$

8.- $\frac{27}{\sqrt[3]{2}} + \frac{24}{\sqrt[3]{16}} + \frac{15}{\sqrt[3]{81}} + \frac{6}{\sqrt[3]{3}}$

9.- $\frac{1}{\sqrt{3}+1} + \frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}-1} + \frac{1}{\sqrt{2}-1}$

10.- $\sqrt[5]{\frac{3a^3b}{2a^2b^3}}; \frac{4\sqrt{3}-3\sqrt{2}}{3\sqrt{3}-2\sqrt{2}}$

11.- $\frac{2}{\sqrt[3]{x}-\sqrt[3]{y}}; \frac{5}{\sqrt[3]{3}+\sqrt[3]{2}}; \frac{6\sqrt{2}}{\sqrt[3]{5}-\sqrt[3]{2}}$

ECUACIONES IRRACIONALES

G) Resolver las siguientes ecuaciones irracionales:

1.- $\sqrt[3]{x+3} = -3$

2.- $2\sqrt{x-7} = 3\sqrt{x-17}$

3.- $a\sqrt[3]{x-b} = b\sqrt[3]{a-x}$

4.- $3\sqrt{2x} - 5\sqrt{8x} + 7\sqrt{18x} = 9\sqrt{20x}$

5.- $2\sqrt{x-7} + 3\sqrt{x+1} = 11\sqrt{x-7}$

6.- $3\sqrt{a-x} + 4\sqrt{x-b} = 4\sqrt{a-x} + 3\sqrt{x-b}$

7.- $2\sqrt{9x+4} - 3\sqrt{4x-11} = 5$

8.- $2\sqrt{x+17} + \sqrt{x+4} = 3\sqrt{x+8}$

9.- $5\sqrt{2x+3} - \sqrt{18x-5} = \frac{4x+12}{\sqrt{2x+3}}$

10.- $\frac{1}{1-x} + \frac{1}{1+\sqrt{x}} + \frac{1}{\sqrt{x}-1} = 0$

11.- $a+x + \sqrt{2ax+x^2} = b$

12.- $\frac{4+x}{\sqrt{x+2}} = 2 - \sqrt{x}$

13.- $\frac{2x-3}{\sqrt{x-2}+1} = 2\sqrt{x-2} - 1$

14.- $3\sqrt{2x-1} - \sqrt{8x+17} = \frac{2(x-3)}{\sqrt{2x-1}}$

15.- $\sqrt{6+4+\sqrt{x^4+10x^2+3x+10}} = x+3$

16.- $\sqrt{3x} - 3\sqrt{2} = \sqrt{2x} - 2\sqrt{3}$

17.- $\frac{3\sqrt{x}-5}{2} - \frac{2\sqrt{x}-7}{3} = \sqrt{x}-1$

18.- $\sqrt{a-x} - \sqrt{b-x} = \frac{a-b}{\sqrt{b-x}}$

Soluciones:

1a) $\sqrt[3]{-8} = \sqrt[3]{-2^3} = -2$

1b) $\sqrt[4]{81} = \sqrt[4]{9^2} = \sqrt{9} = 3$

1c) $\sqrt{-8} = \text{Noreal}$

2a) $\sqrt[4]{3^2} = 3^{2/4} = 3^{1/2} = \sqrt{3}$

2b) $\sqrt{\frac{225}{484}} = \frac{\sqrt{225}}{\sqrt{484}} = \frac{\sqrt{3^2 \cdot 5^2}}{\sqrt{2^2 \cdot 11^2}} = \frac{3 \cdot 5}{2 \cdot 11} = \frac{15}{22}$

2c) $\sqrt[3]{7^3 \cdot 5^6} = 7 \cdot 5^2$

2d) $\sqrt[6]{5^3} \cdot \sqrt[4]{5^7} \cdot \sqrt{2} = \sqrt[12]{5^6} \cdot \sqrt[12]{5^{21}} \cdot \sqrt[12]{2^6} = \sqrt[12]{5^6 \cdot 5^{21} \cdot 2^6} = \sqrt[12]{5^{27} \cdot 2^6} =$

$= \text{podemos simplificar: } 3 = \sqrt[4]{5^9 \cdot 2^2} = \text{extraemos factores} = 5^2 \sqrt[4]{5 \cdot 2^2} = 25 \sqrt[4]{20}$

4a) $7\sqrt{2} + 3\sqrt{2} - 2\sqrt{2} = (7+3-2)\sqrt{2} = 8\sqrt{2}$

4b) $-2\sqrt{7} + 5\sqrt{7} - 8\sqrt{7} + 3\sqrt{7} - 5\sqrt{7} + 7\sqrt{7} = (-2+5-8+3-5+7)\sqrt{7} = 0\sqrt{7} = 0$

4c) $4\sqrt[3]{9} + \frac{1}{2}\sqrt[3]{9} - \frac{3}{4}\sqrt[3]{9} = (4 + \frac{1}{2} - \frac{3}{4})\sqrt[3]{9} = \frac{15}{4}\sqrt[3]{9}$

4d) $\sqrt{18} + 3\sqrt{2} - \sqrt{50} = (1^\circ \text{ factorizams}) = \sqrt{2 \cdot 3^2} + 3\sqrt{2} - \sqrt{2 \cdot 5^2} = (2^\circ \text{ extraemos}) = 3\sqrt{2} + 3\sqrt{2} - 5\sqrt{2} = \sqrt{2}$

6) Desarrollar los siguientes ejercicios con raíces.

1. $\sqrt{4 \cdot 9}, \sqrt[3]{3 \cdot 3 \cdot 9}, 3\sqrt{7} \cdot \sqrt{6} \cdot \sqrt{2}; \sqrt{\frac{2a}{3}} \cdot \sqrt{\frac{3}{4a}}$

$(6; 3; 6\sqrt{21}; \sqrt{\frac{1}{2}})$

2. $\sqrt[3]{a^{2x-y}} \cdot \sqrt[3]{a^{x+y}}; 4\sqrt{a^{3x+2}} \cdot 2\sqrt{a^{2-3x}} \rightarrow (a^x; 8a^2)$

3. $\sqrt[3]{6\sqrt{3}+9}(\sqrt[3]{6\sqrt{3}-9}) + (\sqrt[3]{7+4\sqrt{3}})(\sqrt[3]{7-4\sqrt{3}})$
(4)

4. $(\sqrt{3} + \sqrt{2})^2 + (3 - \sqrt{6})^2 + (\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})$
(21-4\sqrt{6})

14. $\sqrt{6+2\sqrt{5}} - \sqrt{6-2\sqrt{5}})^2 + 3\sqrt{6+2\sqrt{5}} \cdot \sqrt{6-2\sqrt{5}}$
 $\rightarrow (32)$

15. $\sqrt{\frac{2a}{3}} \cdot \sqrt{\frac{3}{8a}} \cdot \sqrt{\frac{2a}{3}} \rightarrow (\sqrt{\frac{a}{6}})$

16. $\sqrt{5+2\sqrt{6}} \cdot \sqrt{5-2\sqrt{6}} + \sqrt[3]{4+2\sqrt{2}} \cdot \sqrt[3]{4-2\sqrt{2}}$ (3)

17. $(\sqrt{a+b+\sqrt{4ab}} - \sqrt{a+b-\sqrt{4ab}})^2 \rightarrow (4b)$

18. $\sqrt[3]{\frac{x-y}{x^2+2xy+y^2}} \cdot \sqrt[3]{\frac{x^2-2xy+y^2}{x+y}} + \frac{4xy}{x^2-y^2}$
 $\rightarrow (\frac{x+y}{x-y})$

19. $(\sqrt{7} + \sqrt{3} + \sqrt{2})(\sqrt{7} - \sqrt{3} + \sqrt{2}) \rightarrow (6+2\sqrt{14})$

20. $(\sqrt{12} + \sqrt{18} + \sqrt{24})(\sqrt{12} + \sqrt{18} - \sqrt{24})$
 $\rightarrow (6+12\sqrt{6})$

21. $(5+2\sqrt{6} + \sqrt{15})(5-2\sqrt{6} + \sqrt{15})$
 $\rightarrow (16+10\sqrt{15})$

22. $\sqrt{\frac{a+b}{a-b}} \cdot \sqrt{a^2-b^2}; \sqrt{6x^2-6} \cdot \sqrt{\frac{3x+3}{2x-2}}$ (a+b;
3x+3)

6) Extraer la raíz de los siguientes productos

$$1. \sqrt{4 \cdot 9}; \sqrt[3]{8 \cdot 27}; \sqrt{a^2 \cdot b^2} \rightarrow (6; 6; ab)$$

$$2. \sqrt{16 \cdot 121 \cdot 400}; \sqrt[3]{8 \cdot 27 \cdot 125} \rightarrow (880; 30)$$

$$3. \sqrt{4a^2} + \sqrt{9a^2} - \sqrt{16a^2} \rightarrow (a)$$

$$4. 4\sqrt[3]{343x^3} - 5\sqrt{36y^2} - 3\sqrt{81x^2} + 6\sqrt[3]{125y^3} (x)$$

$$5. \sqrt{(x^2 - 2xy + y^2)(a+b)^2} \rightarrow (x-y)(a+b)$$

$$6. 3\sqrt{8} + 2\sqrt{32} + 7\sqrt{50} - 6\sqrt{162} + 9\sqrt{98} + 7\sqrt{242} \rightarrow (135\sqrt{2})$$

$$7. 3\sqrt{12a} + 2\sqrt{27a} - \sqrt{75a} \rightarrow (7\sqrt{3a})$$

$$8. \sqrt{15} + \sqrt{20} - \sqrt{80} - \sqrt{125} \rightarrow (\sqrt{15} - 7\sqrt{5})$$

$$9. \sqrt{2} + \sqrt{50} + \sqrt{98} - \sqrt{288} \rightarrow (\sqrt{2})$$

C) DIVISIÓN DE RAICES DE IGUAL ÍNDICE.

$$1. \sqrt[3]{\frac{64}{27}} + \frac{\sqrt{32x^5y^5}}{\sqrt{2x^3y^3}} + \sqrt{\frac{18xy}{25}} : \sqrt{\frac{9xy}{5a^2}}$$

$$\left(\frac{4}{3} + 4xy + a\sqrt{\frac{2}{5}}\right)$$

$$2. \frac{\sqrt{a^x}(\sqrt{a^{x-1}} - \sqrt{a^{3x}})}{\sqrt{a^{1-x}}} \rightarrow (\sqrt{a^{3x-2}} - \sqrt{a^{5x-1}})$$

$$3. \left(\frac{\sqrt{a^2 - b^2}}{\sqrt{a-b}} + \frac{\sqrt{(a+b)^3}}{\sqrt{(a+b)^2}}\right) \rightarrow (2\sqrt{a+b})$$

$$4. (4\sqrt[3]{16} - 6\sqrt[3]{54} + 8\sqrt[3]{128}) : 2\sqrt[3]{2} \rightarrow (11)$$

MISCELANEA

Calcular :

$$1. \frac{5\sqrt{50} + 18\sqrt{32} - 6\sqrt{200}}{3\sqrt{2}} \rightarrow \left(\frac{37}{3}\right)$$

$$2. \sqrt[3]{16} + \sqrt[3]{250} + 2\sqrt[3]{54} \rightarrow (13\sqrt[3]{2})$$

$$3. (3\sqrt{7} + \sqrt{2})^2 - (2\sqrt{7} + \sqrt{2})(2\sqrt{7} - \sqrt{2}) \rightarrow (39 + 6\sqrt{14})$$

$$4. (\sqrt{5} - 1)(\sqrt{2} + 1) - (\sqrt{5} + 1)(\sqrt{2} + 1) + \sqrt{2} \rightarrow (-2 - \sqrt{2})$$

$$5. (2 + \sqrt{2})(2 - \sqrt{3}) - \sqrt{2}(2 - \sqrt{3}) \rightarrow (4 - 2\sqrt{3})$$

$$6. (6\sqrt{7} - 5\sqrt{7} + 2\sqrt{7})(8\sqrt{2} - 5\sqrt{2})\sqrt{2} \rightarrow (18\sqrt{7})$$

$$7. (\sqrt{18} : \sqrt{9} + \sqrt{27} : \sqrt{9} - \sqrt{32} : \sqrt{8} - \sqrt{3} + 2)\sqrt{2} \rightarrow (2)$$

D) Expresa como cociente y calcula:

$$1. \left(\sqrt[3]{\frac{27}{8}} - \sqrt[3]{\frac{125}{64}} + 3\sqrt[3]{\frac{27}{8}}\right) : \left(\sqrt[3]{\frac{64}{27}} + \sqrt[3]{\frac{81}{64}}\right) \rightarrow \left(\frac{114}{59}\right)$$

$$2. \sqrt{\frac{2(a^2 + b^2)^2}{c^2} - \frac{2(a^2 - b^2)^2}{c^2}} \rightarrow \left(\frac{2ab}{c}\sqrt{2}\right)$$

$$3. \sqrt{ab} : \left(\sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}}\right) \rightarrow \left(\frac{ab}{a-b}\right)$$

$$4. \sqrt[3]{\frac{25^3}{64^3}} + \sqrt[3]{\frac{8^2}{27^2}} - \sqrt[3]{\frac{25^x}{125^x}} \rightarrow \left(\frac{83}{120}\right)$$

E) Aplicando la propiedad de la raíz de una raíz calcular:

$$1. 3\sqrt[3]{\sqrt{x}} - 2\sqrt[3]{\sqrt{x}} + 5\sqrt[6]{x} \rightarrow (6\sqrt[6]{x})$$

$$2. \sqrt[3]{\sqrt{a^3}} + \sqrt[5]{\sqrt{a^5}} - 2\sqrt[7]{\sqrt{a^7}} \rightarrow (0)$$

$$3. 2\sqrt[3]{\sqrt{ab}} + \sqrt[6]{\sqrt{ab}} - 4\sqrt[4]{\sqrt{ab}} + \sqrt[6]{\sqrt{ab}} \rightarrow (0)$$

$$4. \sqrt[9]{\sqrt{x^{11}}} \cdot \sqrt[3]{\sqrt{x^{10}}} \cdot \sqrt{x} \rightarrow (\sqrt[30]{x^{29}})$$

$$5. \sqrt[9]{\sqrt{\sqrt{x^6}}} + \sqrt[3]{\sqrt[5]{\sqrt[3]{x^5}}} - \sqrt[3]{\sqrt[4]{\sqrt{x^6}}} - \sqrt[3]{\sqrt[15]{x^5}} \rightarrow (\sqrt[6]{x} - \sqrt[4]{x})$$

$$6. (x+y)\sqrt{\frac{x^2 + y^2}{x^2 + 2xy + y^2}} \rightarrow (\sqrt{x^2 + y^2})$$

$$7. (\sqrt{3} + \sqrt{2})\sqrt{5 - 2\sqrt{6}}; (2 + \sqrt{3})\sqrt{7 - 4\sqrt{3}} \rightarrow (1); (1)$$

$$8.- \left(\frac{9^{\frac{x+1}{4}} \sqrt{9^{\frac{1}{2}} \cdot 3^x}}{3\sqrt{3^{-x}}} \right)^{\frac{1}{x}} ; \left[\frac{(x-1)}{3} \sqrt[3]{\frac{(x-1)^2}{3}} \right]^4$$

$$\rightarrow (3^{3-\frac{1}{x}} ; \sqrt[3]{\frac{(x-1)^{20}}{3^{16}}})$$

$$9.- (2+\sqrt{3})\sqrt{7-4\sqrt{3}} ; (7+\sqrt{5})\sqrt{54-14\sqrt{5}}$$

$$\rightarrow (1) ; (44)$$

F) Aplica la propiedad de cambio de índice

$$1.- \sqrt[3]{a} \cdot \sqrt{a^3} \cdot \sqrt[5]{a} ; x \cdot \sqrt{a^{x^2-y^2}} ; \frac{\sqrt[5]{x^2} \cdot \sqrt[3]{x^4}}{\sqrt[5]{x} \cdot \sqrt[3]{x^5}}$$

$$\rightarrow (\sqrt[30]{a^{61}} ; a^{x+y} ; \sqrt[15]{x^{-2}})$$

$$2.- 2\sqrt{a} - 3\sqrt[3]{a} + 5\sqrt[4]{a} \quad (2\sqrt[12]{a^6} - 3\sqrt[12]{a^3})$$

$$3.- \frac{5\sqrt[3]{a^2b^2} \cdot 2\sqrt[3]{a^2b}}{2\sqrt{ab} \cdot 3\sqrt[4]{a^3b^3}} \quad \left(\frac{5\sqrt[12]{a}}{3\sqrt[4]{b}} \right)$$

$$4.- \sqrt[3]{ab^2} \cdot a\sqrt{ab}\sqrt{ab} \cdot \sqrt[9]{a^5b^3} \rightarrow (a^2b \sqrt[36]{a^{23}b^{27}})$$

$$5.- \left[\frac{a \sqrt[3]{x\sqrt{ax}} \cdot x \cdot \sqrt[5]{a^3\sqrt{a^2x}} \cdot ax \cdot \sqrt[4]{a^3x^2}}{2x} \right]^2$$

$$\rightarrow (a^7 x^5 \sqrt[30]{a^{13}x^6} : 2)$$

$$6.- \frac{a^3 \sqrt{x} \cdot 2x \cdot \sqrt{\sqrt{a^3}}}{\sqrt[3]{x\sqrt{2x^3}} (\sqrt{2a})^3} \rightarrow (a^{\frac{9}{4}} \cdot 2^{\frac{2}{3}} \cdot x^{\frac{2}{3}})$$

$$7.- \frac{a^4 \sqrt{x^5 y^{17}} \cdot \sqrt[4]{b^4 x y^9}}{\sqrt[4]{x^{13} y^9 a^4} \cdot b \cdot \sqrt[4]{x^5 y^5}} \rightarrow \left(\frac{y}{x} \right)^3$$

$$8.- \sqrt{2\sqrt{2^3\sqrt{2\sqrt{2}}}} + \sqrt{3^3\sqrt{3\sqrt{3\sqrt{3}}}} \rightarrow (\sqrt[24]{2^{21}} + \sqrt[24]{3^{19}})$$

H) Racionalizar:

$$1.- \frac{1}{\sqrt{2}} ; \frac{\sqrt{2}}{3\sqrt{3}} ; \frac{5+\sqrt{18}}{\sqrt{2}} \rightarrow \left(\frac{\sqrt{2}}{2} ; \frac{\sqrt{3}}{9} ; \frac{5\sqrt{2}+6}{2} \right)$$

$$2.- \frac{a}{\sqrt[3]{a}} ; \frac{x}{\sqrt[4]{x}} ; \frac{a}{\sqrt{x}} \rightarrow (\sqrt[3]{a^2} ; \sqrt[4]{x^3} ; \sqrt[4]{a^{x-1}})$$

$$3.- \frac{a\sqrt{a}}{\sqrt[3]{a^2}} ; \frac{3}{\sqrt[5]{5}} \rightarrow (\sqrt[6]{a^5} ; \frac{3\sqrt[5]{5^4}}{5})$$

$$4.- \frac{6}{2+\sqrt{2}} ; \frac{1}{3-\sqrt{2}} ; \frac{\sqrt{5}}{\sqrt{5}-\sqrt{2}}$$

$$\rightarrow (6-3\sqrt{2} ; \frac{3+\sqrt{2}}{7} ; \frac{5+\sqrt{10}}{3})$$

$$5.- \frac{\sqrt{2}-\sqrt{3}}{2+\sqrt{2}} ; \frac{\sqrt{2}-\sqrt{2}}{\sqrt{2}+\sqrt{2}} ; \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}$$

$$\rightarrow (2+\sqrt{10}-\sqrt{15}-\sqrt{6} ; \sqrt{2}-1 ; 2-\sqrt{3})$$

$$6.- \left(\frac{2-\sqrt{2}}{1-\sqrt{2}} \right)^{\frac{1}{2}} \rightarrow [-(1+\sqrt{2})\sqrt{4-3\sqrt{2}}]$$

$$7.- \frac{2\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{3\sqrt{2}}{\sqrt{5}-\sqrt{2}} + \frac{2}{\sqrt{7}-\sqrt{5}}$$

$$\rightarrow (5+\sqrt{15}+\sqrt{10}+\sqrt{7}+\sqrt{5})$$

$$8.- \frac{27}{\sqrt[3]{2}} + \frac{24}{\sqrt[3]{16}} + \frac{15}{\sqrt[3]{81}} + \frac{6}{\sqrt[3]{3}} \rightarrow \left(\frac{117\sqrt[3]{4}+22\sqrt[3]{9}}{6} \right)$$

$$9.- \frac{1}{\sqrt{3}+1} + \frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}-1} + \frac{1}{\sqrt{2}-1}$$

$$\rightarrow (2\sqrt{2}+\sqrt{3})$$

$$10.- \sqrt[5]{\frac{3a^3b}{2a^2b^3}} ; \frac{4\sqrt{3}-3\sqrt{2}}{3\sqrt{3}-2\sqrt{2}} \quad \left(\frac{\sqrt[5]{48ab^3}}{2b} ; \frac{24-\sqrt{6}}{19} \right)$$

$$11.- \frac{2}{\sqrt[3]{x}-\sqrt[3]{y}} ; \frac{5}{\sqrt[3]{3}+\sqrt[3]{2}} ; \frac{6\sqrt{2}}{\sqrt[3]{5}-\sqrt[3]{2}}$$

ECUACIONES IRRACIONALES

l) Resolver las siguientes ecuaciones irracionales:

1.- $\sqrt[3]{x+3} = -3 \quad \rightarrow (-30)$

2.- $2\sqrt{x-7} = 3\sqrt{x-17} \quad \rightarrow (25)$

3.- $a\sqrt[3]{x-b} = b\sqrt[3]{a-x} \quad \rightarrow \left(\frac{ab^3 + a^3b}{a^3 + b^3}\right)$

4.- $3\sqrt{2x} - 5\sqrt{8x} + 7\sqrt{18x} = 9\sqrt{20x} \quad \rightarrow (0)$

5.- $2\sqrt{x-7} + 3\sqrt{x+1} = 11\sqrt{x-7} \quad \rightarrow (8)$

6.- $3\sqrt{a-x} + 4\sqrt{x-b} = 4\sqrt{a-x} + 3\sqrt{x-b}$
 $\rightarrow \left(\frac{a+b}{2}\right)$

7.- $2\sqrt{9x+4} - 3\sqrt{4x-11} = 5 \quad \rightarrow (5)$

8.- $2\sqrt{x+17} + \sqrt{x+4} = 3\sqrt{x+8} \quad \rightarrow (8)$

9.- $5\sqrt{2x+3} - \sqrt{18x-5} = \frac{4x+12}{\sqrt{2x+3}} \quad \rightarrow (3)$

10.- $\frac{1}{1-x} + \frac{1}{1+\sqrt{x}} + \frac{1}{\sqrt{x}-1} = 0 \quad \rightarrow \left(\frac{1}{4}\right)$

11.- $a+x + \sqrt{2ax+x^2} = b \quad \rightarrow \frac{(a-b)^2}{2b}$

12.- $\frac{4+x}{\sqrt{x+2}} = 2 - \sqrt{x} \quad \rightarrow (0)$

13.- $\frac{2x-3}{\sqrt{x-2}+1} = 2\sqrt{x-2} - 1 \quad \rightarrow (6)$

14.- $3\sqrt{2x-1} - \sqrt{8x+17} = \frac{2(x-3)}{\sqrt{2x-1}} \quad \rightarrow (13)$

15.- $\sqrt{6+4+\sqrt{x^4+10x^2+3x+10}} = x+3 \quad \rightarrow (5)$

16.- $\sqrt{3x} - 3\sqrt{2} = \sqrt{2x} - 2\sqrt{3} \quad \rightarrow (6)$

17.- $\frac{3\sqrt{x}-5}{2} - \frac{2\sqrt{x}-7}{3} = \sqrt{x}-1 \quad \rightarrow (25)$

18.- $\sqrt{a-x} - \sqrt{b-x} = \frac{a-b}{\sqrt{b-x}} \quad \rightarrow (a)$